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ABSTRACT

Problem solving is viewed as an ability required throughout life, and how we function is seen to be directly related to how well we solve problems. In this document, the issue of a steady decline in students' abilities in the area of mathematical problem solving is confronted and a program designed to teach students the necessary skills in this area is presented. Major goals of the program include building carefully on what students already know, motivating pupils, giving students a successful experience in mathematics, and teaching them to enjoy the accomplishments of thinking a problem through to conclusion. This document presents an overview to the Problem Solving in Mathematics (PSM) project, the project's approach to problem solving, materials from the fifth-grade level packet of supplementary activities, information on research on problem solving in general and on the PSM project, a limited bibliography, and four appendices. (MP)

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Problem Solving in Mathematics

An ESEA Title IV-C Instructional Program
Developed at the
Lane Educational Service District

by
Oscar Schaaf and Richard Brannan

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OSSC BULLETIN

SE 034 926

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Problem Solving in Mathematics

An ESEA Title IV-C Instructional Program
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Lane Educational Service District

by

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Preface

Problem solving is an ability required throughout life. How we function is directly related to how well we solve problems. Finding a job and keeping it, relating to others, coping with inflation, environment, shortages, and a multitude of other problems that beleaguer us indicate clearly how central problem solving is to successful living. It is because of this centrality that educators and board members must be concerned about the growth pupils make in tackling problems.

In recent years national test results have shown a steady decline in students' abilities in the area of mathematical problem-solving. Despite declining scores very little time is devoted either in most texts or in most classrooms to problem-solving activities.

In this *Bulletin*, Oscar Schaaf and Richard Brannan confront this issue and present educators with a program designed to teach students skills necessary to problem solving.

Dr. Schaaf has wide experience as a math educator. He headed the math department at South Eugene High School and has been a mathematics specialist for Lane County, a math coordinator for Eugene School District, and a professor at the University of Oregon since 1954.

Mr. Brannan also has a background of practical teaching experience at the junior high level and as a curriculum writer for the National Science Foundation's Mathematics Resource Project based at the University of Oregon. He is currently a math consultant for Lane County ESD.

Both authors believe that motivating students and building carefully on what students already know are crucial to good math development. Giving students a successful experience in math and teaching them to enjoy the accomplishment of thinking a problem through to conclusion are goals of the program. To achieve these goals, a number of teaching concepts are used, which the authors present here in an easily grasped and practical way.

As further evidence of the excitement this program is generating, we are pleased to announce that the National Science Foundation has awarded it a $30,000 grant to be used through the University of Oregon to prepare leaders for implementing the Mathematics Problem Solving Program in elementary, middle, and junior high schools. A summer workshop is being scheduled now. If you are interested in attending, please contact Oscar Schaaf, College of Education, University of Oregon, Eugene, Oregon 97403.

Philip K. Piele
Executive Secretary
OSSC

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Judy Johnson	Math Consultant, Lane County ESD. Judy has over fifteen years' experience as an elementary teacher.
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Scott McFadden	Math Consultant, Lane County ESD. Teacher, Kennedy Junior High, Eugene. Scott has over twenty years of teaching experience at the junior and senior high levels.
Meredith Schaaf	Secretary, Lane County ESD. Without Meredith this project could not have been completed.

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An Overview

Problem solving has been the subject of research by mathematics educators, educational psychologists, and philosophers since the 1930s. Yet, an examination of the results of national tests, including the recent mathematics tests of the National Assessment of Educational Progress, indicates that students and graduates from our schools have serious deficiencies in their problem-solving abilities. Also, an examination of current mathematics texts and the results of classroom visitations reveal that problem solving is now only a minor part of mathematics instruction in both elementary and secondary schools.

To alleviate this deficiency the Oregon Department of Education ESEA Title IV-C office funded the Problem Solving in Mathematics (PSM) project in 1977. Housed at the Lane Education Service District, the project staff has developed mathematics materials to be used in grades four through nine. Now in its fourth year of funding, the project is essentially finished with the process of development. The project is in the second year of a three-year field test and is preparing for dissemination of materials within Oregon.

Materials from the project are grouped into three-year programs: Program A (elementary school) for grades four through six, Program B (middle school) for grades five through seven (or grades six through eight), and Program C (junior high school) for grades seven through nine. Within each program an individual teacher's packet consists of about eighty lessons emphasizing (1) problem-solving skills early in the year, (2) topics normally taught in a particular grade level, and (3) challenge problems to be used throughout the year after students have been introduced to problem-solving skills. A teacher's commentary accompanies each lesson.

The project's first two years were spent developing and pilot testing pupil materials in selected schools within Lane County. The third year saw the start of the field test of grade four in Program A and grade five in Program B. To be selected as a field-test site, schools responded to announcements in Oregon Department of Education publications as well as in The Oregon Mathematics Teacher, a publication of the Oregon Council of Teachers of Mathematics. To the staff of each responding school, project personnel made an awareness presentation, sharing materials and discussing requirements for being involved. A dual acceptance--the school staff for the project and the project staff for the school--was necessary to be involved in the field test. A list of field-test schools is found in Appendix A.

Based on the results of the first year of the field test, Programs A and B were certified by the Advisory Council for ESEA Title IV-C. Program C, now in its first year of field test, is expected to receive certification in 1981. Certification allows any school district in Oregon to make application to the project for using the materials. Adoption grants are also available to help defray costs of the implementation. See Appendix C for an announcement appearing in Promising Practices in Oregon Education, published by the Oregon Department of Education.

More information is available from the project or from the Title IV-C office.

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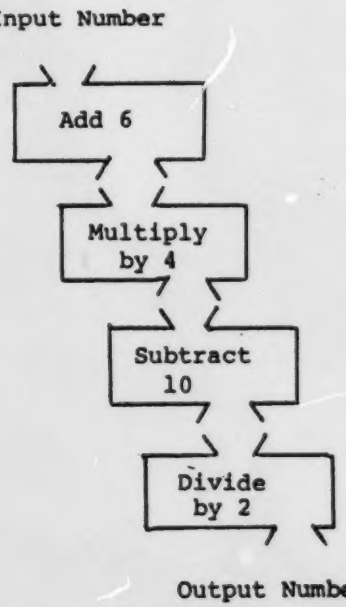
The Project's Approach to Problem Solving

The assumption was made that all learners can improve their ability to use problem-solving skills if given the opportunity and motivation to do so. Problem solving can be taught if teachers are provided with suitable pupil materials, if they have developed an expertise in teaching the problem-solving process, and if problem solving is the focus of instruction. The following information is organized around key questions that might be asked by educators who might be interested in adopting PSM.

A Problem: What Is It?



	Input Number	Output Number
a.	4	
b.	8	
c.	12	
d.		39
e.		47
f.		61

- Suppose a 6th-grader were asked to fill in the missing output blanks for a,b, and c in the table. Would this be a problem for him? Probably not, since all he would need to do is to follow the directions.

◉ Suppose a 2nd-year algebra student were asked to fill in the missing input blank for d. Would this be a problem? Probably not, since she would write the suggested equation,

$$\frac{4(x+6) - 10)}{2} = 39$$

and then solve it for the input.

◉ Now suppose the 6th-grader were asked to fill in the input for d; would this be a problem for him? Probably it would be. He has no directions for getting the answer. However, if he has the desire, it is within his power to find the answer. What might he do? Here are some possibilities:

1. He might make guesses, do checking, and then make refinements until he gets the answer.
2. He might fill in the output numbers that correspond to the input numbers for a,b, and c.

	Input	Output
a.	4	15
b.	8	23
c.	12	31
d.		39

and then observe this pattern:

For an increase of 4 for the input,

the output is increased by 8.

Such an observation should lead quickly to the required input of 16.

3. He might start with the output and work backwards through the machine hook-up using the inverse (or opposite operations).

For this pupil, there was no "ready-made" way for him to find the answer, but most motivated 6th-grade pupils would find a way.

> A problem, then, is a situation in which an individual or group accepts the challenge of performing a task for which there is no immediately obvious way to determine a solution. Frequently, the problem can be approached in many ways. Occasionally, the resulting investigations are nonproductive. Sometimes they are so productive as to lead to many different solutions or suggest more problems than they solve.

What Does Problem Solving Involve?

Problem solving requires the use of many skills. Usually these skills need to be used in certain combinations before a problem is solved. A combination of skills used in working toward the solution of a problem is called a strategy. A successful strategy requires the individual or group to generate the information needed for solving the problem. A considerable amount of creativity can be involved in generating this information. Thus, problem-solving strategies and creativity are closely related.

What Are Some Examples of Problem-Solving Skills?

Skills are the building blocks used in solving a problem. The pupil materials developed by this project afford many opportunities to emphasize problem-solving skills. A listing of some of these skills is given below. Some skills are unique to mathematics; others are interdisciplinary.

- Clarify the problem by asking questions
- Make and use a drawing or model
- Make a systematic list or table
- Break a problem into manageable parts
- Use mathematics symbols to describe situations
- Look for patterns
- Make predictions
- Make a reasonable estimate
- Guess and check
- Change a problem into one you can solve (simplify the problem)

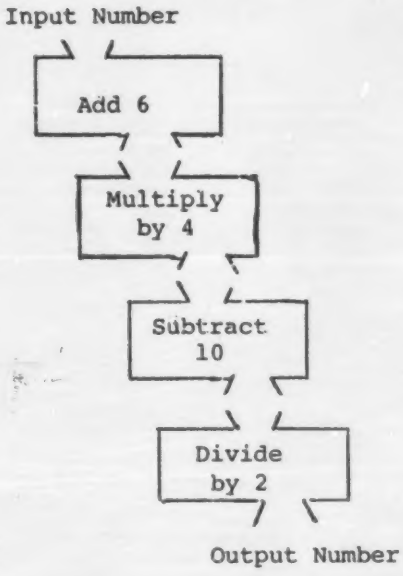
- Work backwards
- Eliminate possibilities
- Find another answer when more than one is possible
- Create new problems by varying one that has been solved

What Are Some Examples of Problem-Solving Strategies?

Since strategies are a combination of skills, a listing (if it were possible) would be even more cumbersome than the list of skills. We will confine our examples of strategies tc those used in the problem from page 4.

Compute and record the output numbers for the first three given input numbers (a,b,c entries in the table). Different strategies could be used to find the input for the d entry in the table. Three possibilities follow:

Strategy 1. Guess the input, check by computing the output number for your guess; if guess does not give the desired output, note the direction of error; refine the guess; compute; continue making refinements until the correct output results.



Strategy 2. Observe the patterns suggested by the input and output numbers for the a,b,c, entries in the table; predict additional output and input numbers by extending both patterns; check the predicted input for the d entry by computing.

Strategy 3. Study the operations suggested in the machine hook-up; work backwards through the machine using previous knowledge about inverse operations.

An awareness of the strategies being used to solve a problem is probably the most important step in the development of a pupil's problem-solving abilities.

Why Teach Problem Solving?

Problem solving is an ability people need throughout life. Pupils have many problems with varying degrees of complexity. Problems arise as they attempt to understand concepts, see relationships, acquire skills, and get along with their peers, parents, and teachers. Adults have problems, many of which are associated with making a living, coping with the energy crisis, living in a nation with peoples from different cultural backgrounds, and preserving the environment. Since problems are so central to living, educators need to be concerned about the growth their pupils make in tackling problems.

Problem solving has been a primary stated goal of mathematics instruction since the turn of the century. The recent National Assessment of Educational Progress (NAEP) has shown problem solving (word problems) to be a weak area for pupils at all three levels tested--9 year olds, 13 year olds, and 17 year olds.

What Is the Project's Instructional Approach to Problem Solving?

The content objectives of the lessons are similar to those of most textbooks. The difference is in the approach used. A wider variety of problem-solving skills is emphasized in the materials than in most texts. Direct instruction, guided discovery, laboratory work, small-group discussions,

nondirective instruction, and individual work all have a role to play in problem-solving instruction.

Most texts employ almost exclusively direct instruction, whereas the lessons in PSM are patterned after a guided discovery approach. Also, an attempt is made in the project materials to use intuitive approaches extensively before teaching formal algorithms. Each of the following methods is an integral part of the instructional approach to problem solving.

A. TEACH PROBLEM-SOLVING SKILLS DIRECTLY

Teachers do emphasize computation and measurement. Also, problem-solving skills such as "follow directions," "listen," and "correct errors" are skills teachers expect pupils to master. Yet such skills as "guess and check," "make a systematic list," "look for a pattern," or "change a problem into one you can solve" are seldom made the object of direct instruction. These skills, as well as many more, need emphasis. Detailed examples for teaching these skills early in the school year are given in the commentaries to the Getting Started activities.

B. INCORPORATE A PROBLEM-SOLVING EMPHASIS IN

- *Drill and practice activities*

  Each problem-solving packet includes one or more sections on drill and practice at the problem-solving level. These sections, along with the Getting Started section, are easy for pupils and teachers to get into and should be started early in the school year.

- *Laboratory activities and investigations involving mathematical applications and readiness activities*

  One way this is accomplished is to use geometric notions to introduce arithmetic ideas, such as using area concepts for the multiplication and division algorithms and fraction concepts.

- *Teaching mathematical concepts, generalizations, and processes*

  Each packet includes two or more sections on content topics such as fractions, decimals, percent, two-digit multiplications, equation solving, etc. For the most part these topics are developmental in nature and need to be followed with practice pages selected from the text.

C. PROVIDE MANY OPPORTUNITIES FOR PUPILS TO USE THEIR OWN PROBLEM-SOLVING STRATEGIES

One section of each grade-level packet includes a collection of challenge activities; in addition, others are scattered throughout other sections. These activities provide opportunities for emphasizing problem-solving strategies. Generally, instruction should be nondirective, but at times suggestions may need to be given. If possible, these suggestions should be made in the form of alternatives to be explored rather than hints to be followed. A specific example on the teaching of these strategies is given in the commentary for the section entitled Challenges, and in the teacher comments for each challenge.

D. CREATE A CLASSROOM ATMOSPHERE IN WHICH OPENNESS AND CREATIVITY CAN OCCUR

Such classroom climate should develop if the considerations mentioned above in A, B, and C are followed. Some specific suggestions to keep in mind as the materials are used are:

- Set an example by solving problems and by sharing these experiences with the pupils.
- Reduce anxiety by encouraging communication and cooperation. On frequent occasions problems might be investigated using a cooperative mode of instruction along with brainstorming sessions.
- Encourage pupils in their efforts to solve a problem by indicating that their strategies are worth trying and by providing them with sufficient time to investigate the problem; stress the value of the procedures pupils use.

- Use pupils' ideas (including their mistakes) in solving problems and developing lessons.
- Ask probing questions which make use of words and phrases such as

  I wonder if -
  Do you suppose that -
  What happens if -
  How could we find out -
  Is it possible that -

- Reinforce the asking of probing questions by pupils as they search for increased understanding. Pupils seldom are skilled at seeking probing questions but they can be taught to do so. If instruction is successful, questions of the type, "What should I do now?," will be addressed to themselves rather than to the teacher.

How Will These Problem-Solving Materials Fit into the Regular Mathematics Program?

The materials developed by the Problem Solving in Mathematics project are not meant to be a complete curriculum. But the topics in the PSM grade-level packets are those commonly included in commercial texts. A textbook correlation chart in each grade-level packet shows how each lesson can be used along with the text. For example, a problem-solving lesson on multiplication would be used in place of all or parts of a multiplication drill and practice page from the text.

The next few pages show selected lessons from a grade-level packet to indicate how the PSM lessons address certain topics normally taught at that grade level. On occasion the teacher commentary for the lesson is included.

PROBLEM SOLVING IN MATHEMATICS

(Fifth-grade Packet)

I. Getting Started

Problems are included for the teaching of different problem-solving skills. These are suggested for the beginning of the year to introduce pupils to problem solving. A direct mode of instruction is recommended.

Guess, check, and refine.

1. a. I am thinking of a number. Two times the number and then add 11 equals 39. What is the number?
   b. I am thinking of a number. Subtract 5 from the number and then add 18 and the answer is 22. What is the number?
   c. I am thinking of a number. The number added to itself and then subtract 13 equals 37. What is the number?

Look for a pattern.

1. Write the next three numbers in each sequence.
   a. 2, 4, 6, ___, ___, ___
   b. 1, 4, 7, 10, ___, ___, ___
   c. 1, 2, 4, 8, 16, ___, ___, ___
   d. 1, 2, 4, 7, 11, 16, ___, ___, ___

Make a systematic list.

5.

   The coins shown above are the only ones you have. What amounts can you make if you use 1, 2, 3, or 4 of the coins?

Make a drawing or use a model.

5. Two hunters came to a river. They met three youngsters who had a small boat. The boat could hold two youngsters or one adult. How did all five persons get across the river? How many trips will be necessary?

II. Whole Number Drill and Practice

No sequence is implied by the order of activities included in this section. They can be used throughout the year but are often appropriate at the beginning of the year when the initial chapter in text emphasizes review.

IV-C:5
Wh.No. D & P

THE OUTERS AND THE INNERS

1. Carlos told his teacher about a discovery he had made. Ms. Harris thought others should try it. See if you can find the discovery.
   a. Use four consecutive numbers like 3, 4, 5, 6.
   b. Find the product of the inside numbers. 4 x 5 = 20
   c. Find the product of the outside numbers. 3 x 6 = 18
   d. Try other numbers.

Numbers	Inside Product	Outside Product
3, 4, 5, 6		
5, 6, 7, 8		
9,10,11,12		
15,16,17,18		
50,51,52,53		

→ Make up your own numbers.

Discovery: _____

2. Debbie wondered about using four consecutive even numbers like 4, 6, 8, 10. Try five cases.

Discovery: _____

Numbers	Inside Product	Outside Product

3. Lonnie thought something different would happen if four consecutive odd numbers like 7, 9, 11, 13 were used. Try five cases.

Discovery: _____

Numbers	Inside Product	Outside Product

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Teacher Commentary for The Outers and The Inners

IV-C:5
Wh.No. D & P
T.C.

The Outers and the Inners (Page 4)

Mathematics teaching objectives:

- Develop mathematics vocabulary such as product, even, odd, consecutive.
- Practice multiplication skills.

Problem solving skills pupils might use:

- Make and use a table.
- Look for and use patterns.
- Create new problems by varying an old one.

Materials needed:

- Calculators (optional)

Comments and suggestions:

- The activity provides drill and practice in multiplication of whole numbers. A calculator could be used as motivation.
- Each pupil needs a copy of the activity. Introducing the activity by using a transparency may be helpful.

Answers:

1.

	Inside Product	Outside Product
3, 4, 5, 6	20	18
5, 6, 7, 8	42	40
9,10,11,12	110	108
15,16,17,18	272	270
50,51,52,53	2652	2650

Discovery: The inside product is 2 greater than the outside product.

2. For four even consecutive numbers, the inside product is 8 greater than the outside product.

3. For four odd consecutive numbers, the inside product is 8 greater than the outside product.

Extension:

Pupils could investigate other patterns such as the inside and outside products for four consecutive multiples of 3, 4, etc.

III. Story Problems

These lessons are excellent supplements for the story problems in the text since they emphasize problem-solving skills different from those usually found in texts.

IV-C:5
Story Prob.

WRITING SHORT STORIES

Calvin got 100% on his story problem lesson.
His work is shown below.
Can you think of story problems that match his work?
Write your problems in the space provided.

CALVIN'S WORK	YOUR STORY PROBLEMS
① 73 + 29 My answer is 102 newspapers.	
② 9 x $1.98 My answer is $17.82.	
③ $100 - $73.95 My answer is $26.05.	
④ 253 − 194 My answer is 59 miles.	
⑤ 7⟌$8.57 My answer is $1.22.	
⑥ $.29 + .49 + .39 My answer is $1.17.	
⑦ 5⟌865 The average weight is 173 pounds.	
⑧ 25 x 12 My answer is 300 eggs.	
⑨ 3 x $.59 + 5 x $.75 My answer is $5.52.	
⑩ 8⟌40 My answer is 5 cookies.	

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Writing Short Stories (Page 3)

Mathematics teaching objective:

. Create single and multiple step word problems.

Problem solving skills pupils might use:

. Search printed material for needed information.

. Create a problem which has been solved by a set of calculations.

Materials needed:

. None

Comments and suggestions:

. Students are often asked to translate real life situations into math symbols. Here is an opportunity to do the reverse. Again, the focus is on understanding the problem.

. The activity provides each pupil a chance to express his/her creativity. Some will create the minimum amount while others will produce an intricate problem often involving unnecessary information.

. Problem 7 involves an average weight which may need to be explained. For those pupils not understanding the concept of average, you could use this problem as an example.

Five large bags of potatoes hold 160, 185, 182, 178 and 170 pounds of potatoes. Sally has the job of moving potatoes from bag to bag until each bag has the same number of pounds of potatoes. How many pounds will each bag have?

Answers:

Answers will vary.

IV. Fractions

A circle model for fractions is used, and the lessons should be done in sequence. We recommend that this section be used as a basis of a fraction teaching unit, with the text being used in a supplementary fashion. It is best to work for the mastery of a region model like circles before doing much work with other models like sets or the number line. Operations with fractions are limited to a manipulative approach.

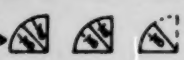
IV-C:5
Fractions

STILL MORE GROUPS

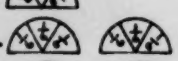
Needed: Circle fraction pieces

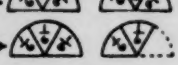
1. a. 2 groups of $\frac{2}{8}$ → = _____

   b. 3 groups of $\frac{2}{8}$ → = _____

   c. $2\frac{1}{2}$ groups of $\frac{2}{8}$ →  = _____

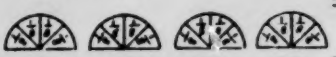
2. a. 1 group of 3 sixths → = _____

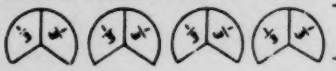
   b. 2 groups of 3 sixths →  = _____

   c. $1\frac{2}{3}$ groups of 3 sixths →  = _____

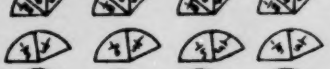
   d. Shade $1\frac{2}{3}$ group of 3 sixths in (c).

How Much?

3. a. Shade $1\frac{3}{4}$ groups of $\frac{4}{8}$.  _____

   b. Shade $2\frac{1}{2}$ groups of $\frac{2}{3}$.  _____

   c. Shade $3\frac{1}{3}$ groups of $\frac{3}{8}$. _____

   d. Shade 3 groups of $\frac{2}{5}$.  _____

   e. Shade $2\frac{4}{5}$ groups of $\frac{5}{6}$. _____

4. Use the fraction pieces.

How Much?

   a. Make $2\frac{1}{2}$ groups of 4 twelfths. _____

   b. Make $1\frac{1}{3}$ groups of 3 fifths. _____

   c. Make $2\frac{1}{2}$ groups of $\frac{4}{8}$. _____

   d. Make $1\frac{3}{4}$ groups of $\frac{4}{6}$. _____

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Teacher Commentary for Still More Groups

Still More Groups (Page 14)

Mathematics teaching objectives:

- Understand concept of multiplication with fractions. (Readiness)
- Multiply fraction by a mixed fraction. (Readiness)

Problem-solving skills pupils might use:

- Use a model.
- Use a drawing.
- Solve an easier but related problem.

Defer the identification of the problems on the pupil pages as examples of multiplication until a later time.

Materials needed:

- Circle fraction pieces.

Comments and suggestions:

- This activity combines the two concepts from the previous activities. Finding $2\frac{1}{2}$ groups of $\frac{2}{8}$ is done by finding 2 groups of $\frac{2}{8}$ and then $\frac{1}{2}$ of a group of $\frac{2}{8}$ for a total of $\frac{5}{8}$. The approach actually is an application of the distributive property, although pupils need not know this. $1\frac{1}{4} \times 4 = (1 \times 4) + (\frac{1}{4} \times 4)$.

Answers:

1. a. $\frac{4}{8}$ b. $\frac{6}{8}$ c. $\frac{5}{8}$
2. a. $\frac{3}{6}$ b. $\frac{6}{6}$ c. $\frac{5}{6}$
3. a. $\frac{7}{8}$ b. $\frac{5}{3}$ c. $\frac{10}{8}$ d. $\frac{6}{5}$ e. $\frac{14}{6}$
4. a. 10 twelfths. b. 4 fifths c. $\frac{10}{8}$ d. $\frac{7}{6}$

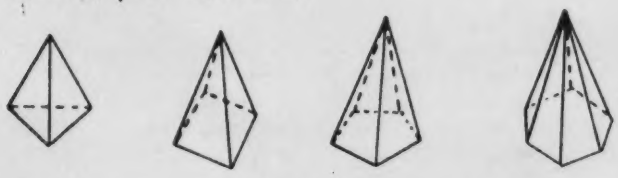
V. Geometry

The lessons have few prerequisites and therefore can be done any time during the year. Most of these lessons deal with "global" properties of geometric objects often not covered in textbooks.

IV-C:5
Geom.

PYRAMIDS

Pyramids have a point, faces that are triangles and bottoms that can have different shapes. A pyramid gets its name from the shape of the bottom.



Triangular Square Pentagonal Hexagonal

1. Use the drawings to help you fill in the table.

Pyramid Name	Number of sides in the bottom	Number of faces including the bottom	Number of corners	Number of edges
Triangular				6
Square		5		
Pentagonal	5			
Hexagonal			7	

2. Answer these by looking for patterns in the table.
   If the bottom of a pyramid has
   a. 8 sides, it has _____ faces.
   b. 13 sides, it has _____ corners.
   c. 50 sides, it has _____ edges.

3. Look for other patterns in the table. Write them on the back of this paper.

4. Find a pattern using the faces, corners, and edges.

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Teacher Commentary for Pyramids

Pyramids (Page 5)

Mathematics teaching objectives:

- Find and use number patterns.
- Relate pictures of solids to models.

Problem solving skills pupils might use:

- Use a drawing.
- Make a systematic listing.
- Observe patterns and then make a prediction.

Materials needed:

- A model for each of the pyramids should be available.

Comments and suggestions:

- This activity can be done as a whole group. Each pupil will need an activity sheet. The vocabulary (sides, faces, edges, corners) can be explained as the models are shown.
- Some pupils will have trouble interpreting the drawings, especially the hidden faces and edges. Actual models will help. Students can match the models to the drawings before starting on the activity. If students have made the models themselves, they will probably have a better understanding of the solids and the drawings.
- A similar activity could be written using prisms--triangular, rectangular, pentagonal, etc. A pattern will occur in the table although it will be different than the pattern for the pyramids.

Answers:

1.

Triangular	3	4	4	6
Square	4	5	5	8
Pentagonal	5	6	6	10
Hexagonal	6	7	7	12

2. a. 9
   b. 14
   c. 100

3. Several patterns will be seen by pupils. Encourage open discussion of whatever patterns they see.

4. Number of corners + number of faces = number of edges + 2. This pattern works for most 3-dimensional objects with "flat" faces.

VI. Decimals

This decimal unit is developed without the use of fractions as a prerequisite. Decimals are introduced with a region model (a grid) followed by money and number line models. The lessons should be done in sequence and be used as the basis for initial work with decimals, with the text being used in a supplementary fashion.

IV-C:5
Dec.

MAKING CENTS OUT OF MONEY

Money--dollars and cents--is a good application of decimals. You should be accurate with calculations but it is also important to be a good estimator.

Ben is helping his dad do the shopping. Estimate to see if they can buy these five items. They have only $10.00.

1 dozen eggs	$ .89	cheese	$1.99	ham	$3.95
lunch meat	$1.07	milk	$1.59		

Do you think they can buy these five items? _____

Ben thought this way:
$.89 is almost $1.00, so
$ .89 → $ 1.00
$1.07 → $ 1.00
$3.95 → $ 4.00
$1.99 → $ 2.00
$1.59 → $ 2.00
$10.00 should be enough

Now you be the best shopper. Write your estimates in the blanks. Then circle the best estimate.

1. $1.89 $3.93 $2.89 $3.79
_____ _____ _____ _____
$9.00 $11.00 $13.00

2. $3.09 $1.89 $.49 $2.61
_____ _____ _____ _____
$6.00 $7.00 $8.00

3. $2.11 $3.14 $1.09 $4.19
_____ _____ _____ _____
$10.00 $11.00 $14.00

4. $.83 $1.19 $4.09 $2.87 2 of these
_____ _____ _____ _____
$8.00 $10.00 $12.00

5. 2 of these $5.89 $2.98 3 of these
_____ _____
$16.00 $19.00 $21.00

6. $5.79 $3.09 4 of these
_____ _____
$8.00 $18.00 $22.00

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Teacher Commentary for Making Cents Out of Money

Making Cents Out of Money (Page 12)

Mathematics teaching objectives:

- Use decimals in solving money problems.
- Estimate money amounts using rounding procedures.

Problem-solving skills pupils might use:

- Make reasonable estimates.

Materials needed:

- None

Comments and suggestions;

- Many pupils would rather compute the answer exactly than estimate. The fear of being wrong inhibits them. A positive class atmosphere and a series of activities in estimation might help. The section on Estimation With Calculators can be used.
- This activity gives pupils practice in either rounding up or rounding down to a convenient amount and then using those amounts to get an estimate. Other activities are important and needed to emphasize the importance of estimation, especially while shopping.
- After all estimates have been made, pupils may want to find the exact answer to see how close it is to the estimate. They can do this by hand or with a calculator. Insisting that they check every estimate with computation might reduce pupil motivation to estimate.

Answers:

1. $13.00
2. $8.00
3. $10.00 Although $10.00 is the estimate most pupils will get for this problem, $11.00 is closer to the actual answer. Discuss how rounding down for each price tag caused the estimate to be be inaccurate.
4. $12.00
5. $21.00
6. $18.00

VII. Probability

These laboratory-oriented lessons provide an opportunity to make and test conjectures. They also give an introduction to probability concepts. The lessons need not be done in sequence.

WHICH DO YOU THINK WILL BE LARGER?

1. Two players need three regular dice. Player A gets two dice. Player B gets the other one.
2. Do this activity.
   a. Player A rolls both dice and finds the product of the two numbers.
   b. Player B rolls the one die and multiplies the number by itself.
   c. The winner is the player with the largest answer.

   Who do you think will win more often? _____
3. Do the activity 30 times. Record the results in a table showing three things: A's answer, B's answer, and the winner.

Number	1	2	3	4	5	6	7	8	9	10
A's Answer										
B's Answer										
Winner										

Number	11	12	13	14	15	16	17	18	19	20
A's Answer										
B's Answer										
Winner										

Number	21	22	23	24	25	26	27	28	29	30
A's Answer										
B's Answer										
Winner										

EXTENSION

Do the activity this way:

1. Player A adds the two numbers.
2. Player B adds the number to itself.

Now who do you think will win more often? _____

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Which Do You Think Will Be Larger? (Page 3)

Mathematical teaching objectives:

- Develop probability concepts.
- Practice basic multiplication facts.
- Practice recording skills.

Problem solving skills pupils might use:

- Collect data needed to solve the problem.
- Use a physical model.
- Make and use a table.

Materials needed:

- 3 regular dice per pair of pupils

Comments and suggestions:

- The activity can be presented (or summarized) with questions such as this:

  What is the problem? To decide who will win most often.
  How can we solve it? By trying it (use a model and collect data).
  How can we keep track of the data? Make a table.
  How can we use the data? To make better guesses (make predictions).

- Have pupils do just one trial and record the winner. Pupils can decide what to do on ties, either re-roll or just count it as a tie.
- Although totals will be close, player B should win more often than player A. For the extension, the totals should be nearly equal.
- The use of homemade foam dice will make this a quieter activity.

Answers:

Answers will vary. One such trial of 30 rolls gave these results:

Player A	3	10	1	4	30	20	30	18	9	4	5	4	6	12	10
Player B	36	16	1	1	25	25	25	4	1	36	1	1	4	25	4
Winner	B	B	-	A	A	B	A	A	A	B	A	A	A	B	A

Player A	3	10	5	10	12	8	6	15	5	8	6	12	8	4	4
Player B	36	16	1	9	4	4	1	36	15	36	1	36	25	9	1
Winner	B	B	A	A	A	A	A	B	B	B	A	B	B	B	A

B won 13 times. A won 16 times. There was one tie.

VIII. Estimation with Calculators

The lessons need not be done in sequence. The calculator is used as a motivator for making estimates and doing mental arithmetic. If calculators are not available, many of these still can be used.

A NUMBER TIMES ITSELF

Use a calculator. Fill in the table.

14 x 14	
24 x 24	
34 x 34	
44 x 44	
54 x 54	
64 x 64	
74 x 74	
84 x 84	
94 x 94	

Now solve these problems. In each case the number in the two boxes must be the same.

1. ☐ x ☐ = 1225
2. ☐ x ☐ = 361
3. ☐ x ☐ = 3249
4. ☐ x ☐ = 4225
5. ☐ x ☐ = 784
6. ☐ x ☐ = 9216
7. ☐ x ☐ = 6724
8. ☐ x ☐ = 11025

EXTENSION

☐ x ☐ x ☐ = 17576

The number in each box must be the same.

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Teacher Commentary for A Number Times Itself

A Number Times Itself (Page 5)

Mathematics teaching objectives:

- Estimate.
- Use a calculator.

Problem solving skills pupils might use:

- Make and use a table.
- Recognize limits and eliminate possibilities.
- Make reasonable estimates as answers.
- Guess, check and refine.

Materials needed:

- Calculator

Comments and suggestions:

- The chart at the top could be filled in as a whole-group activity. This would allow you to determine if all pupils were correctly using the calculator. There is no intended pattern in the products except that all end in a 6.
- The remainder of the sheet should be an individual (or pair) task. Observe the pupils using the guess, check and refine strategy. Perhaps a hint or two about using the chart will help.
- Pupils will use two distinct problem solving strategies after the table has been completed. Some will finish the bottom by guessing, using the calculator to check, and then refining the guess. Others will use the data in the table, recognize limits, and apply what they know about multiplication facts to arrive at the answer.
- The entire activity will take about 25 minutes.

Answers:

Chart: 196; 576; 1156; 1936; 2916; 4096; 5476; 7056; 8836

1. 35	5. 28
2. 19	6. 96
3. 57	7. 82
4. 65	8. 103

Extension: 26

IX. Challenges

Challenges are included for use throughout the year after completion of Section I and part of Section II. A nondirective instructional mode is recommended.

IV-C:5
Chal.

A LOGICAL CHOICE

1. Larry, Curly, and Moe left the theater together. Each one was wearing the jacket and the shoes of someone else.

   Curly was wearing Moe's jacket and Larry's shoes.

   Whose jacket and whose shoes was each one wearing?

2. a. A yuk is more than an ugh.
   b. A bah is not the least of the whole group.
   c. A yuk is not the greatest.
   d. Only one thing is less than an ick.
   e. A glok is more than a bah.
   f. More than one thing is greater than a bah.

   Put the yuk, ugh, bah, ick, and glok in order from largest to smallest.

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Teacher Commentary for A Logical Choice

IV-C:5
Chal.
T.C.

A Logical Choice (Page 4)

Problem-solving skills pupils might use:

- Break a problem into manageable parts.
- Make and use a table.
- Eliminate possibilities using contradictions.
- Guess, check, and refine.

Comments and suggestions:

- Logic problems lend themselves to using a table. List one variable across the top. Use of yes and no, or X and O, or any convenient notation permits the problem solver to use each clue to find the answer.

	Larry	Curly	Moe
Jacket			
Shoes			

- Students can solve the second problem by guessing and checking. Some may write the names on slips of paper and move them around to check various arrangements. A table could also be used to solve this problem. Squares are X'd out if it is not a possible position. The bah can't be least. The ick is second, so it is written in and the rest of it's row and column are X'd out. (No one else can be second.) etc.

	least →			most
yuk		X		
ugh		X		
bah	X	X		
ick	X	ick	X	X
glok		X		

Answers:

1.

	Larry	Curly	Moe
Jacket		Moe's	
Shoes		Larry's	

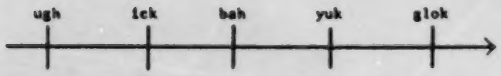
(Given)

	Larry	Curly	Moe
Jacket	Curly's	Moe's	Larry's
Shoes	Moe's	Larry's	Curly's

Larry cannot wear his own jacket so he must wear Curly's.

Moe cannot wear his won shoes so he must wear Curly's

2.



Research on Problem Solving in General

An article, "Untangling Clues from Research in Problem Solving," by Marilyn N. Suydam, in *Problem Solving in School Mathematics*, the 1980 Yearbook of the National Council of Teachers of Mathematics, summarized the recent research into mathematical problem solving. Several excerpts pertinent to the Problem Solving in Mathematics Project are listed here. The author and year are included below, but each separate item does not appear in the references at the end of this document. The reader is referred to the article for a more complete treatment and for a very extensive bibliography.

> Experience and some research suggest that certain problem-solving skills which will improve mathematical problem-solving performance can be learned--provided the teacher illustrates how the procedure works; gives ample opportunity for discussion, practice, and reflection; and supports and encourages the learner's efforts.
> (Kilpatrick 1978)

> Training in a variety of problem-solving skills is necessary so that students have a repertoire from which they can draw as they meet the wide variety of problems that exist.
> (Brandau and Dorsey 1979)

> When problem-solving skills are specifically taught, they are then used more and students achieve correct solutions more frequently.
> (Clement, et al. 1977)

> Allow children sufficient time to solve problems. Not all problems can be solved within the confines of one mathematics lesson; some need to be 'mulled over' for longer periods of time.
> (Suydam 1980)

...practice in problem solving should not consist in repeated experiences in solving the same problems with the same techniques, but should consist of the solution of different problems by the same techniques and the application of different techniques to the same problems.
(Brownell 1942)

Manipulative materials, pictures, diagrams, and similar aids enhance the probability of a problem's being understood and solved correctly.
(Ammon, et al. 1973)

Problem-solving skills are improved by incorporating them throughout the curriculum--...
(Roman 1975)

Do not be misled by the immediate benefits of structural [expository] teaching--the long-range benefits of teaching through problem solving may be higher.
(Flener 1978)

Instead of being 'protected' from error, the child should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why.
(Brownell 1942)

Research on the PSM Project

A dissertation by Al Peacock (1979) involved twelve students and their problem-solving behaviors. All students had exposure to the problem-solving materials because their respective teachers were part of a pilot test of the materials. Several conclusions from the study follow:

- The verbal and written protocols of this study, and subjects' sentiments in regard to problem solving, indicate that subjects were using the problem-solving skills taught by their mathematics teachers, with purpose and direction.

- Subjects gave indications of emotional as well as intellectual involvement while problem solving. Their feelings were transmitted both verbally and visually. Elation, disappointment, enthusiasm, relief, interest and determination were some of the most overt indications of emotional involvement.

- On several occasions subjects referred to problem solving as being an opportunity to "use my brain." If suitable problems can be devised and if teachers could introduce mathematical problem solving skills as a part of the regular mathematics curriculum, then math could become "both fun and a learning experience."

At this time none of the programs has been field tested for the full three-year period. Program A, the elementary school program, and Program B, the middle school program, are now in their second field-test year. The junior high program, Program C, is just now in its first year (1980-81). However, both summative and formative data were collected last year on both Programs A and B. The summative data included results from two tests developed by the project -- Problem Solving Skills Test (PSST) and the Problem Preference Test (PPT)--and from the 1978 Metropolitan Mathematics Survey Tests (MAT).

Formative data were collected as a result of classroom visitations and interviews with pupils, teachers, and principals.

The conclusions that follow were taken from a report (Dizney 1980) prepared by the independent evaluators after the first year of field testing.

> As may be seen from the data (PSST) substantial Fall and Spring gains accrued to the Field School pupils at both 4th and 5th grade levels. The differences were not only statistically significant for total groups at each grade level, but underlying these general results were significant improvements for each of the seven fourth grade classes and six of the nine fifth grade classes.
>
> The data for the Metro test for both fourth and fifth grades reveal that overall the students grew at a somewhat faster rate than is normal for students at their grade levels. If a student maintains his/her position relative to his/her peers during the course of the school year it is interpreted as normal growth. Three of the fourth grade classes failed to maintain that level of growth as evidenced by the metro test results whereas only one fifth grade class experienced a slight loss which may easily be attributed to measurement error because it was only two percentile rank points. Thus, four out of seven fourth grade classes and seven out of nine fifth grade classes improved their standings relative to the nationally normed instrument.

In addition to the study referred to above, a graduate student made a special study (Mayes 1980) of the first-year use of the fourth-grade packet (Program A). His conclusions concerning the extent to which problem-solving skills were learned are:

- The findings of this study support the contention that overall, students learned problem-solving skills to a satisfactory extent for the first year of field testing.

- The findings of this study support the contention that all individual field-test classrooms learned problem-solving skills to a satisfactory extent for the first year of field testing.

- Findings from the PPT support the contention that overall, students increased their "liking" of problem-solving type problems and their willingness to tackle such problems. This conclusion was corroborated by descriptive findings.

- Findings from the MAT support the contention that overall, the development of basic numeracy skills by students was not negatively affected by interaction with the materials. In fact, descriptive findings tend to support the contention that emphasis on problem-solving skills may have enhanced the development of basic numeracy skills.

Many other conclusions were made by Mayes relating to variations in teacher effectiveness in the use of the materials, inservice education provisions, and the effect the materials had on the teacher's teaching style. Mayes reached the following conclusion concerning the reaction of students:

> Overall, according to field-test teachers, their students enjoyed working with the materials and gained considerably from their interaction with them. Observations and student interviews confirmed this point of view. All of the students interviewed over the field-test year stated that they liked the problem-solving materials. No negative or disparaging remarks about the materials were heard directly from individual students during observations. Most students appeared to have an awareness of the four basic problem-solving skills.

An independent study (Elliott 1980) was conducted in three different Alaskan school districts to determine the effectiveness of the calculator

along with selected sixth-grade problem-solving materials in improving verbal problem-solving performance of sixth graders.

The problem-solving sections of the intermediate and advanced mathematics forms on the 1970 Metropolitan Achievement Tests (MAT) were given as a pre-test to the classes. For seven weeks teachers provided problem-solving and calculator instruction. Two post-tests were given, one test where calculators were used and the other where they were not.

The gain shown on both tests was substantially (statistically significant) more than was predicted for a full year's growth at the sixth-grade level. According to the MAT norm tables, the expected gain is 3.50 items. The gain in the test without the calculator was 4.41 items over the seven-week period; with the calculator, 6.64 items.

Near the end of the first year of the field test, teachers were interviewed. The following are summaries of the comments.

- All felt pupils liked the project materials. Many reacted very favorably when they recognized the distinctive format and typing on a reproduced lesson sheet.

- Several said their style of teaching was positively influenced by the materials. One went from a teacher-centered approach to a cooperative, less formal approach. Another became more inclined to form small groups to facilitate a cooperative learning situation and to encourage inquiry-oriented activities. An experienced teacher who already uses many resources said the materials provided him with new ideas and different ways to present content. Another felt he now had materials to fill a need he had been unable to deal with previously. Finally, one teacher said her teaching style was unaffected by the materials because she already had an inquiry/discovery emphasis but the materials certainly provided more opportunities for the inquiry/discovery to happen.

- An unemphasized goal of the problem-solving approach to mathematics education is the transfer of problem-solving skills to other curriculum areas. Several teachers reported seeing no transfer. Others reported transfer to science and social studies. Examples of pupils using the skills of "look for a pattern," "make a systematic list," and "guess, check, and refine," were mentioned. One teacher thought her pupils were assisted by the problem-solving materials to improve their word attack skills and their level of awareness of sequencing in language. Another felt the cooperative mode of instruction emphasized throughout the materials had helped with the social development of her pupils. Finally, one teacher felt her pupils had come to appreciate that there may be more than one correct answer to a problem and that there is often more than one way to solve a problem.

A release from the News Bureau at the University of Oregon prompted a correspondent from the Oregonian in Portland to visit Sweetbriar Elementary School (Troutdale) to observe pupils involved in problem solving activities. Interviews that followed brought these comments from teachers, principal, and the project's inservice consultant.

"...gives students the tools to solve problems beyond their math skills. We teach kids it's all right to guess because they can learn from wrong answers if they check them out. They can refine their answers until they hit the right one."

"My kids generally like it. It's a fun approach and I enjoy teaching it."

"We're trying to pull kids away from the idea that for a given problem, there is only one answer. There can be many ways to solve a problem and several answers to just one problem. We're teaching this now because as adults, these kids will need multiple ways to find solutions."

"...gets kids out of thinking math is just working exercises in textbooks. Problem solving math challenges students into thinking rather than just memorizing."

Summary

During the first field-test year, several difficulties were encountered. Teachers themselves need to feel comfortable with problem solving. The fidelity to the instructional approach advocated by the PSM staff was not as consistent as desired. Integration of the materials into the regular curriculum was not as much as the staff wanted. Some teachers used the materials more in a supplementary fashion. Not much time was spent discussing the skills and strategies used by students on a particular activity. And finally, students were not given enough time to mull and think about a problem. Observations and interviews with teachers indicate a lessening of these difficulties during the second year of use.

Despite these difficulties the use of the Problem-Solving in Mathematics materials holds much promise. Gains are being made in problem-solving abilities. Standardized mathematics test scores are not negatively affected. In fact, significant gains were made. The project materials are filling a need expressed by several groups within the mathematics education community. And students and teachers genuinely enjoy using the materials!

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Elliott, James W. The Effect of Using Hand-Held Calculators on Verbal Problem Solving Ability of Sixth Grade Students. Unpublished doctoral dissertation, University of Oregon, August 1980.

Krulik, Stephen and Reys, Robert E., Editors, NCTM Yearbook. PROBLEM SOLVING IN SCHOOL MATHEMATICS. 1980.

Mayes, Leslie. An Evaluative Study of the First Year of an Elementary School Mathematics Problem-Solving Program. Unpublished doctoral dissertation, University of Oregon, December 1980.

Peacock, Alistair A. Analysis of Process Sequence Traces Observed in Mathematical Problem Solving. Unpublished doctoral dissertation, University of Oregon, September 1979.

Appendix A

FIELD TEST SCHOOLS

Alsea Elementary School, Alsea, OR

Lincoln Elementary School, Ashland, OR

Estacada Junior High School, Estacada, OR

St. Alice Catholic School, Eugene, OR

Glendale Elementary School, Glendale, OR

Deer Creek Elementary School, Glide, OR

Talmadge Junior High School, Independence, OR

Monroe Elementary School, Monroe, OR

North Plains Elementary School, North Plains, OR

May Roberts Elementary School, Ontario, OR

Philomath Middle School, Philomath, OR

Riverdale Elementary School, Portland, OR

Obsidian Junior High School, Redmond, OR

Salem Heights Elementary School, Salem, OR

Scott Elementary School, Salem, OR

Sweetbriar Elementary School, Troutdale, OR

Warrenton Elementary School, Warrenton, OR

Judson Junior High School, Salem, OR

Sprague High School, Salem, OR

Appendix B

OTHER SCHOOLS USING THE PSM MATERIALS

Elementary Schools

Canyonville Elementary School, Canyonville, OR

Elmira Elementary School, Elmira, OR

Coburg Elementary School, Eugene, OR

Condon/Magnet Arts Elementary School, Eugene, OR

Crest Drive Elementary School, Eugene, OR

Dunn Elementary School, Eugene, OR

Edgewood/Evergreen Elementary School, Eugene, OR

Edison/Eastside Elementary School, Eugene, OR

Howard Elementary School, Eugene, OR

Laurel Hill Elementary School, Eugene, OR

Lincoln Elementary School, Eugene, OR

Magladry Elementary School, Eugene, OR

McCornack Elementary School, Eugene, OR

Meadow Lark Elementary School, Eugene, OR

O'Hara School, Eugene, OR

Parker Elementary School, Eugene, OR

St. Paul School, Eugene, OR

Silver Lea Elementary School, Eugene, OR

Spring Creek Elementary School, Eugene, OR

Twin Oaks Elementary School, Eugene, OR

Washington Elementary School, Eugene, OR

Willagillespie Elementary School, Eugene, OR

Willard Elementary School, Eugene, OR

Irving Elementary School, Eugene, OR

McKenzie River Elementary, Finn Rock, OR

Harrisburg Elementary School, Harrisburg, OR

Pleasant Hill Elementary School, Pleasant Hill, OR

Centennial Elementary School, Springfield, OR

Goshen Elementary School, Springfield, OR

Guy Lee Elementary School, Springfield, OR

Junior High or Middle Schools

Cascade Middle School, Eugene, OR

Crolane Junior High, Eugene, OR

Jefferson Junior High, Eugene, OR

Kelly Junior High, Eugene, OR

Kennedy Junior High, Eugene, OR

Madison Junior High, Eugene, OR

Monroe Junior High, Eugene, OR

Roosevelt Junior High, Eugene, OR

Shasta Middle School, Eugene, OR

Spencer Butte Junior High, Eugene, OR

Cal Young Junior High, Eugene, OR

Harrisburg Junior High, Harrisburg, OR

Hedrick Junior High, Medford, OR

McLoughlin Junior High, Medford, OR

Westridge School, Oakridge, OR

Springfield Middle School, Springfield, OR

Hamlin Middle School, Springfield, OR

Senior High Schools

Elmira High School, Elmira, OR

McKenzie River High School, Finn Rock, OR

Harrisburg High School, Harrisburg, OR

Churchill High School, Eugene, OR

North Eugene High School, Eugene, OR

Sheldon High School, Eugene, OR

South Eugene High School, Eugene, OR

Oakridge High School, Oakridge, OR

Appendix C
Project Announcement

PROBLEM SOLVING IN MATHEMATICS

(Appearing in Promising Practices in Oregon Education,
Oregon Department of Education)

Program A: Elementary Schools - Grades 4-6

Program B: Middle Schools - Grades 5-7

Program C: Junior High Schools - Grades 7-9 (ready in 82-83)

Description: Mathematics materials designed to improve the problem-solving ability of pupils in upper elementary, middle, and junior high schools. The materials teach basic computational skills and problem-solving skills and are written to integrate with Oregon adopted textbooks. All basic skill areas are covered. Problem solving pages replace some or all of some textbook pages. A textbook correlation chart is included with the teacher materials. The intent is to use problem solving for about one-third of the mathematics instruction. Each three-year program includes four components: (1) pupil materials, (2) teacher commentaries including mathematics objectives, problem-solving skills, materials needed, and answers, (3) consultant services, and (4) an evaluation plan.

Student Outcomes: After one year of use in field-test schools in grades 4 and 5, pupils made significant gains in problem-solving skills as measured by an instrument developed by the project staff, and in general mathematics achievement as measured by the mathematics survey test of the Metropolitan Achievement Tests. Results of testing in grade 7 will be available after May of 1981.

Developed at the Lane Education Service District, the materials are being tested in over fifty elementary and middle schools within Lane County. Outside the county, the first year of programs A and B has been replicated with summative and/or formative evaluation in schools in Ashland, Glendale, Glide, Monroe, Ontario, Philomath, Salem, Troutdale, and Warrenton. The first year of Program C is being replicated in schools in Alsea, Estacada, Independence, Redmond, Riverdale, and Salem.

Target Population: All pupils, grades 4-7 (4-9 available in 82-83 school year)

Instructional Methods: A wide variety of instructional methods are recommended for different parts of the programs, including the direct instruction of certain problem-solving skills, the developmental instruction of mathematical topics, laboratory approaches, and a largely non-directive approach to challenge activities.

Facility Requirements: No alterations are required.

Materials, Equipment Requirements: The project will provide adopters with an adopter's guide, a master copy of pupil and teacher materials for each involved teacher appropriate for his/her grade level and a master copy of the evaluation instruments developed by the project staff at a cost of about $15 per teacher. Extra manipulatives required are those items normally found in most classrooms. A list of such items is provided. The school district may need to purchase some items that are not available. A district must reproduce pupil materials at its own expense. Estimates for ditto reproduction are three reams of paper, forty ditto masters, and one-fourth of a can of ditto fluid per teacher using the materials.

Personnel: No increase in personnel is required.

Training: Five inservice sessions of about one hour each (usually held after school) will be conducted throughout the school year. The inservice sessions will be conducted by mathematics leaders trained and familiar with the programs. These consultants could assist teachers in integrating the programs into the regular mathematics curriculum. Costs of the in-service, including honorarium, travel and expenses, should be about $500 per school.

A summer workshop on implementation of the materials will be held at the University of Oregon. Districts willing to send a representative will receive a higher priority for acceptance into the program. Expenses for the participant are included within available adoption grants. Graduate credit of 4-6 hours may be earned.

Adoption agreement: Adopting districts must contact either Oscar Schaaf or Richard Brannan (Lane ESD, P.O. Box 2680, Eugene, OR 97402; telephone 689-6500, extensions 41 or 75 or 17) to work out an adoption plan and agreement. Districts should know the project wants all mathematics teachers in the three grade levels of adopting schools to be involved.

Appendix D

PROBLEM-SOLVING PROGRAMS - A BRIEF OVERVIEW

The Lane County Mathematics Project staff firmly believes that pupils will become better problem solvers if they learn a variety of problem-solving skills and have many opportunities to apply the skills in activities both in and outside of the classroom. The following instructional approach is being used effectively to accomplish the above objective.

1. Problem-solving skills are taught directly. Using direct instruction, problem-solving skills such as "guess and check," "look for a pattern," "make a list," etc., are taught early in the school year. Ten to fifteen minutes per day for about one month are spent working on problems specifically written to highlight particular problem-solving skills.

2. Problem-solving activities to be used throughout the school year are provided in each of the following areas:

   a. drill and practice activities
   b. laboratory activities and investigations
   c. mathematical concept development

3. Non-routine or challenge problems provide opportunities for pupils to use their problem-solving skills. The creative meshing together of several problem-solving skills to form a solution strategy is the most crucial phase of problem solving. The teacher's role is nondirective. Being sure pupils understand the problem and asking probing questions are the two important functions of teachers.

Effective instruction in mathematics problem solving needs an atmosphere in which openness and creativeity can occur. The above approach fosters this atmosphere. A more rapid development happens if the teacher also

sets an example by actually solving problems,

encourages communication and cooperation with the problem-solving process,

uses pupil ideas in solving problems.